Dynamicalization: Adaptive Manipulation of Constraints for Efficient Evolutionary Constrained Optimization

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Introduction

- An introduction to constrained optimization
- Basic idea and first approach
- Simulation results
- Summary and outlook

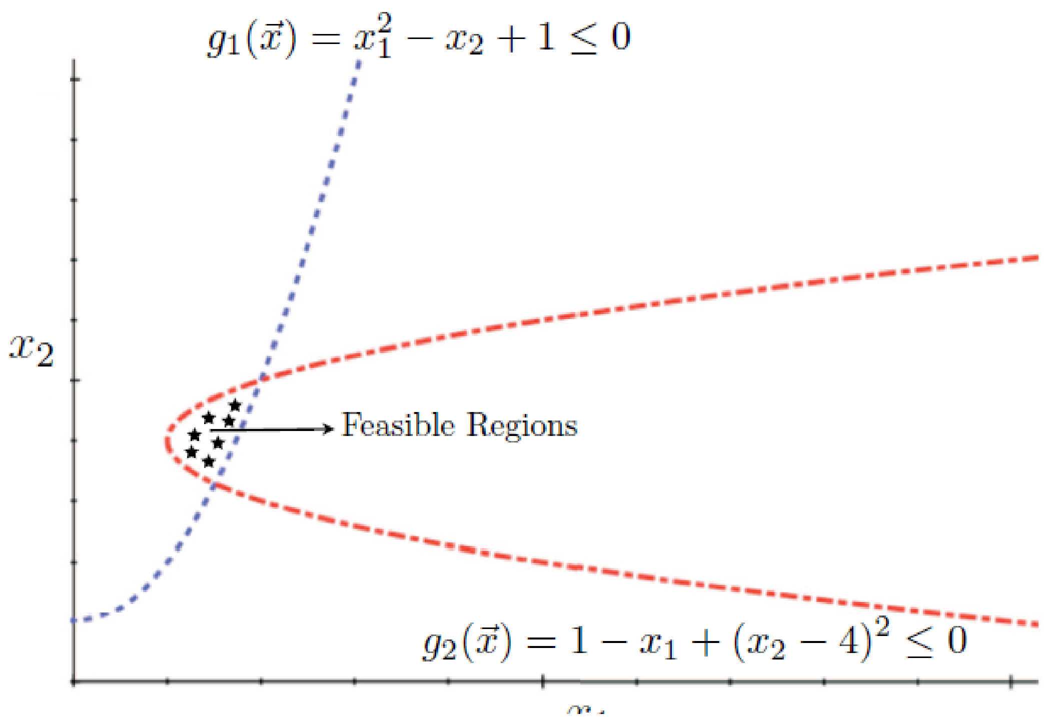
Constrained Optimization

$$\text{Minimize } f(\vec{x})$$ $\vec{x}$ is vector of solutions $\vec{x} = [x_1, x_2, \cdots, x_n]^T$

subject to:

$$g_i(\vec{x}) \leq 0,\ i = 1, \cdots, m$$ m is the number of inequality constraints

$$h_j(\vec{x}) = 0,\ j = 1, \cdots, p$$ p is the number of equality constraints



Existing Methods for Handling Constraints

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- Penalty functions

Constrained optimization problems

Minimize $f(\vec{x})$
subject to:

$$g_i(\vec{x}) \leq 0,\ i = 1, \cdots, m$$
$$h_j(\vec{x}) = 0,\ j = 1, \cdots, p$$

Transformation

Unconstrained optimization problems

$$\phi(\vec{x}) = f(\vec{x}) \pm \left[\sum_{i=1}^{m} r_i \times G_i + \sum_{j=1}^{p} c_j \times H_j \right]$$

$G_i = max[0, g_i(\vec{x})]^\beta$, where is $\beta = 1$

$H_j = |h_j(\vec{x})|^\gamma$, where is $\gamma = 2$

○ Penalty factor

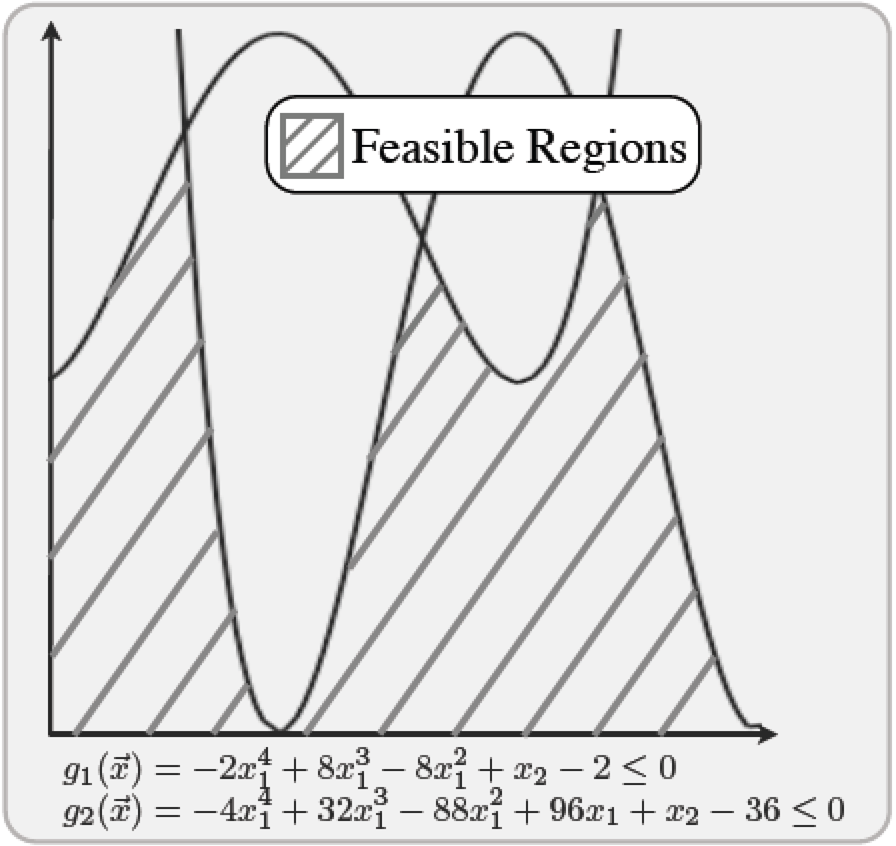
- How to determine the penalty factors?
  – death penalty, stationary, dynamic, adaptive

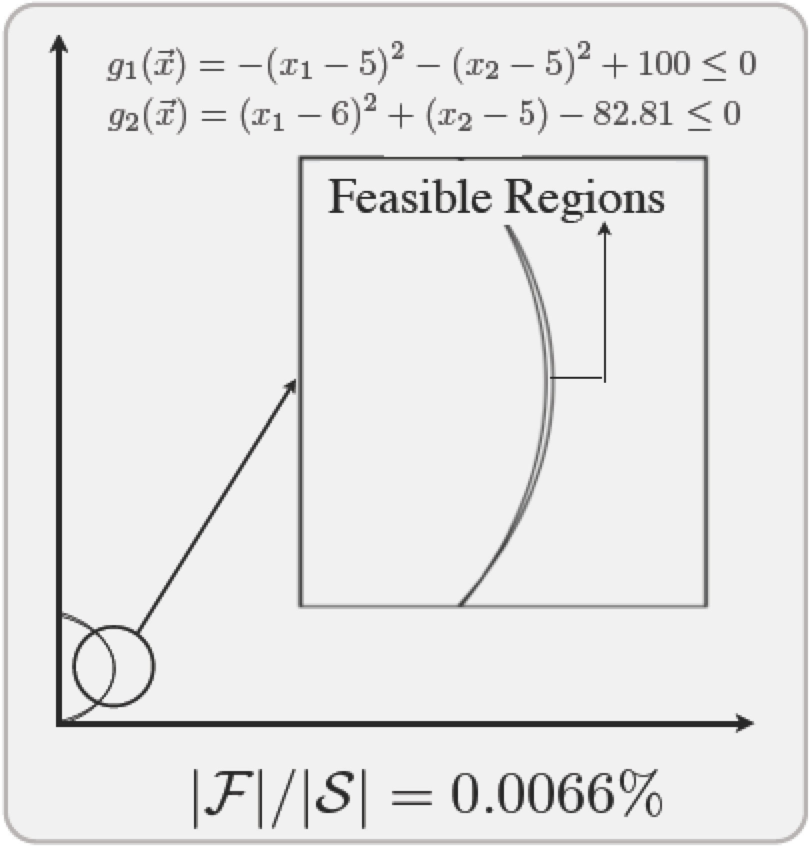
Existing Methods for Handling Constraints

- Repairing
- Stochastic ranking
- Multi-objectivization
- …

Highly Constrained Optimization Problems

- The feasible regions may be small and isolated

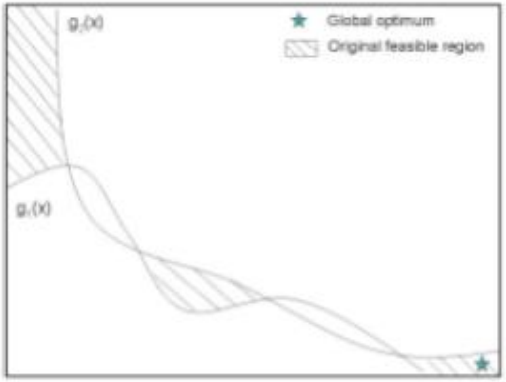




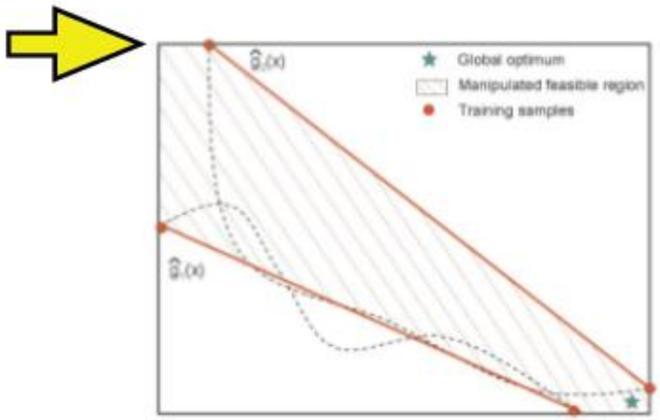
Basic Ideas

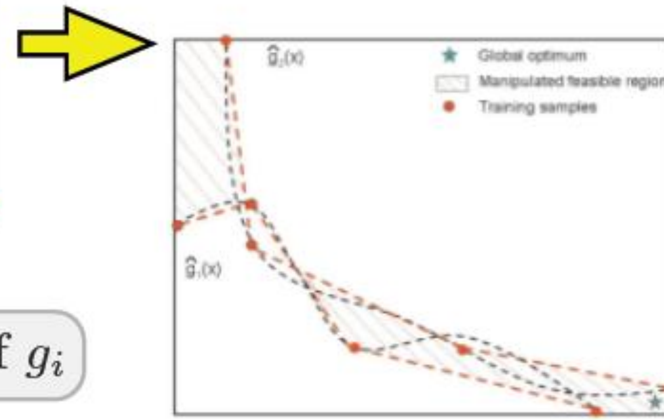
- To manipulated the feasible region by changing the constraint functions
  - The manipulated feasible region should be much larger than the real one, and it should converge to the real one gradually
  - The adaptive manipulation of the feasible region is realized by manipulating the constraints
  - An approximate model is built for each constraint functions, whose complexity increase as the evolution proceeds

Illustration of the Basic Idea



Utilize approximation model to synthetically enlarge feasible regions.





$\hat{g}_i$: Approximation of Constraint of g_i

- Achieving incremental approximation accuracy by training a model using increasing data samples of the constraint function

How to Use the Manipulated Constraints

- Use synthesized constraints that van be a mixed combination of the real and approximate constraints
- Choose the combination by maximizing the degree of feasibility of the current population

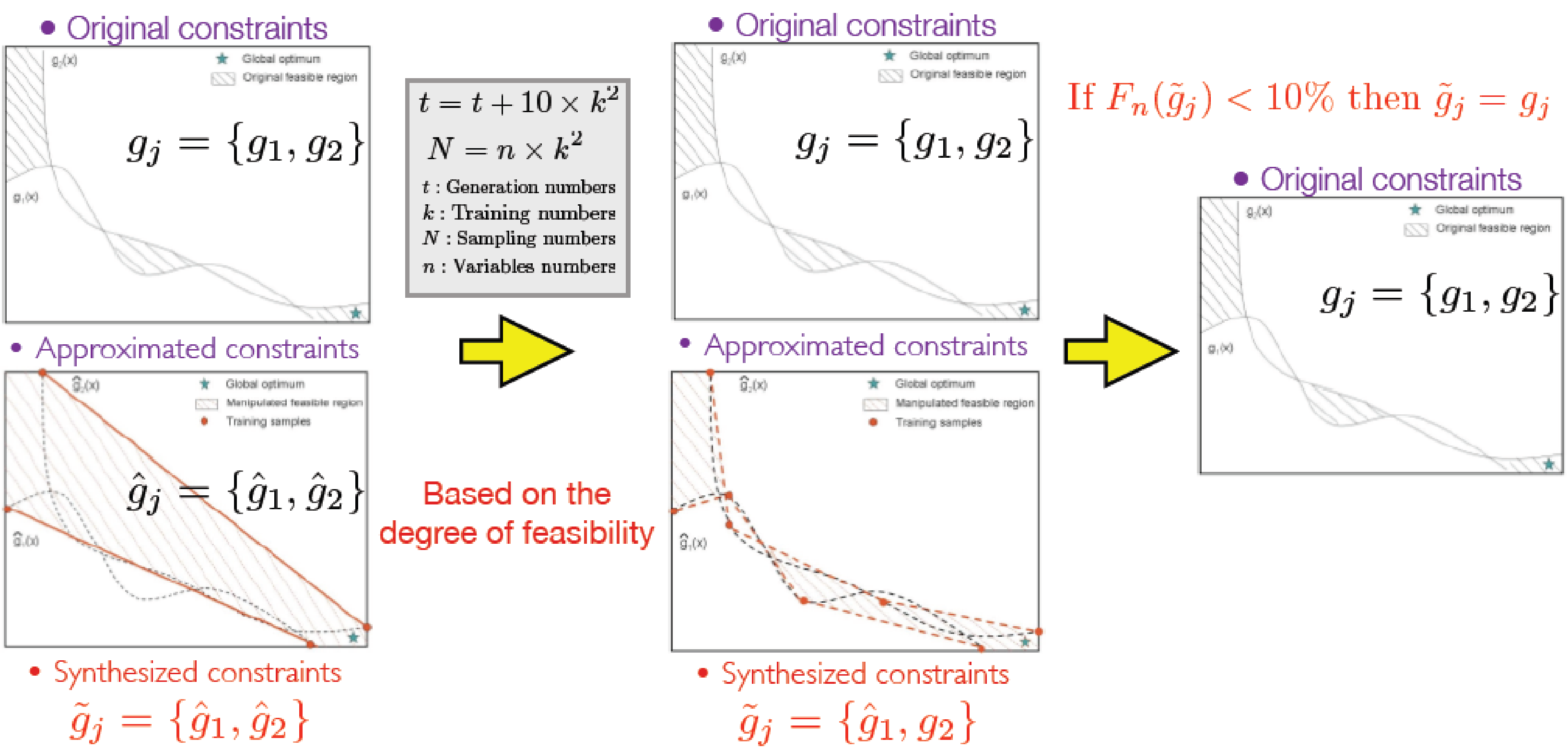
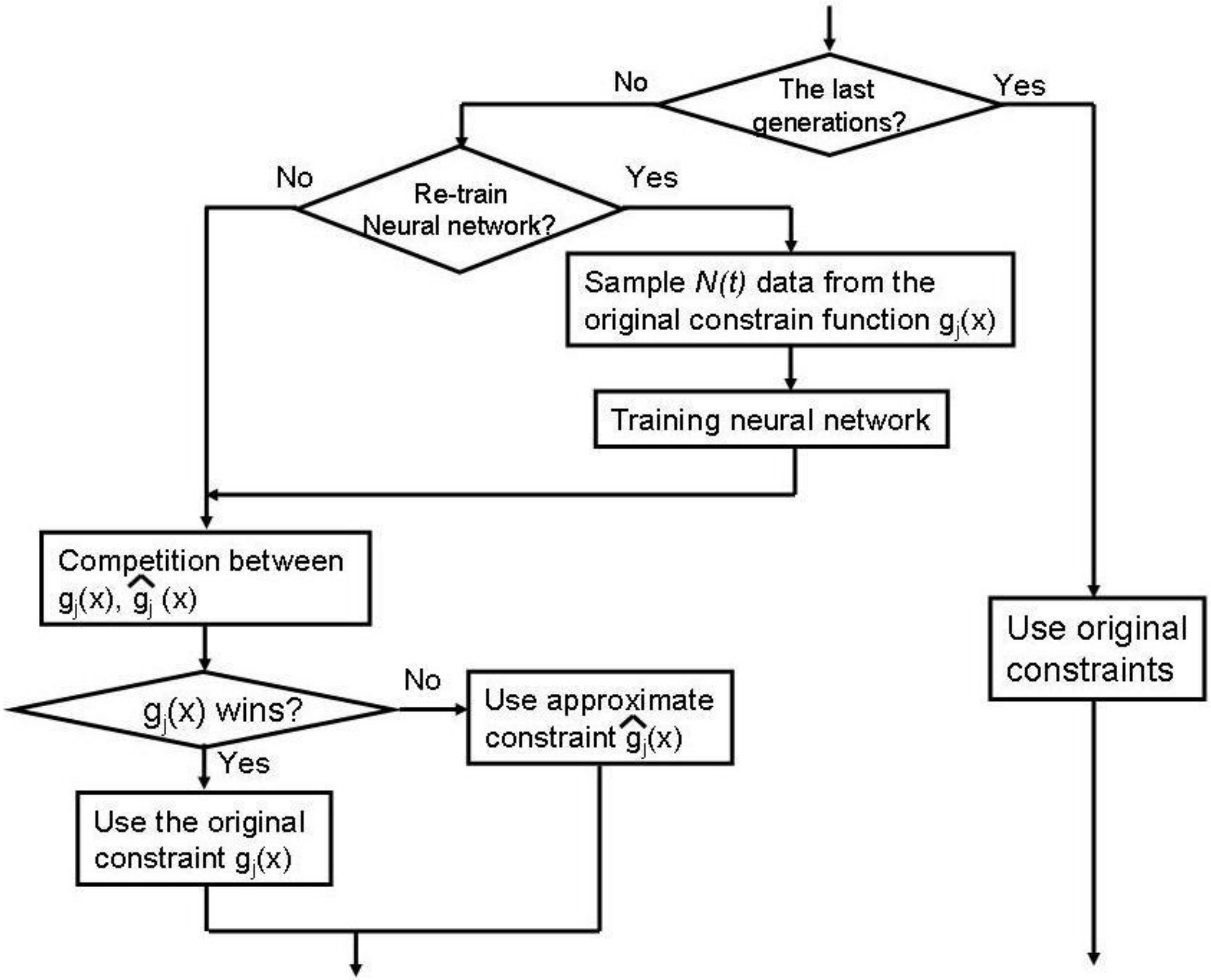
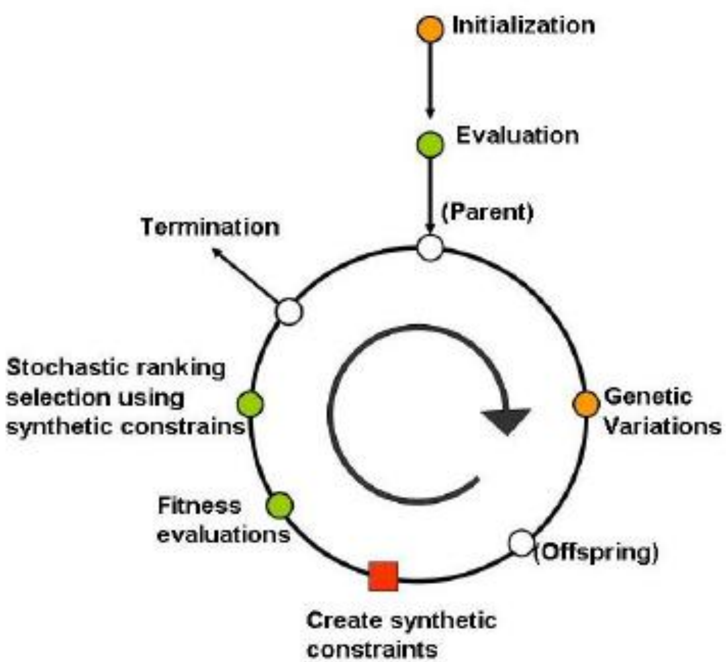


Diagram of the Manipulation Algorithm



The Algorithm

- On top of the Stochastic Ranking Evolution Strategy (SRES) (Runarsson and Yao, 2000)
- Using synthetic constrains; from generation 911 on, only original constraints are used
- The neural network is updated in generations 0, 10, 50, 140, 300, 550, 910; Number of samples: Nj , 4Nj , 9Nj , 16Nj , 25Nj , 36Nj , 49Nj; Nj (>=2) is the dimension
- 30 independent runs



Stochastic Ranking

for i=1 to λ, **do**
  for j=1 to $\lambda - 1$, **do**
    sample $u \in U(0,1)$, U is a uniform distribution
    if($\tilde{\phi}_j = \tilde{\phi}_{j+1} = 0$) **or** ($u < 0.45$), **then**
     if ($f_i > f_j$), **then**
      swap the order of individual i and j
     fi
    fi
  od
  if no swap done **break** **fi**
od

T.P. Runarsson and X. Yao, “Stochastic ranking for constrained evolutionary optimization”, *IEEE Trans. on Evolutionary Computation*, 4(3):284-294, 2000.

Comparative Studies

	ATMES	SRES	SMES	SRES-SC
Parent size	50	30	100	30
Offspring size	300	200	300	200
Generations	800	1750	800	1200
Evaluations	240000	350000	240000	240000

- ATMES: Yong Wang, Zixing Cai, Yuren Zhou and Wei Zeng, "An adaptive tradeoff model for constrained evolutionary optimization," IEEE Transactions on Evolutionary Computation, vol. 12, pp. 80-92, Feb. 2008.
- SRES: Thomas P. Runarsson and Xin Yao, "Stochastic ranking for constrained evolutionary optimization," IEEE Transaction on Evolutionary Computation, vol. 4, pp. 284-294, Sep. 2000.
- SMES: Efren Mezura-Montes and Carlos A. Coello Coello, "A simple multimembered evolution strategy to solve constrained optimization problems," IEEE Transaction on Evolutionary Computation, vol. 9, pp. 1-17, Feb. 2005.

Test Problems

Prob.	n	type of function	$\lvert F\rvert/\lvert S\rvert$	LI	NI	LE	NE	a
g01	13	quadratic	0.0111%	9	0	0	0	6
g02	20	nonlinear	99.9971%	0	2	0	0	1
g03	10	polynomial	0.0000%	0	0	0	1	1
g04	5	quadratic	52.1230%	0	6	0	0	2
g05	4	cubic	0.0000%	2	0	0	3	3
g06	2	cubic	0.0066%	0	2	0	0	2
g07	10	quadratic	0.0003%	3	5	0	0	6
g08	2	nonlinear	0.8560%	0	2	0	0	0
g09	7	polynomial	0.5121%	0	4	0	0	2
g10	8	linear	0.0010%	3	3	0	0	6
g11	2	quadratic	0.0000%	0	0	0	1	1
g12	3	quadratic	4.7713%	0	1	0	0	0
g13	5	nonlinear	0.0000%	0	0	0	3	3

T.P. Runarsson and X. Yao, “Stochastic ranking for constrained evolutionary optimization”, *IEEE Transactions on Evolutionary Computation*, 4(3):284-294, 2000.

Results on the Test Problems - Mean

	ATMES	SRES	SMES	SRES-SC
g01	**-15**	**-15**	**-15**	**-15**
g02	-0.790148	-0.781975	-0.785238	**-0.792114**
g03	**-1**	**-1**	**-1**	**-1**
g04	**-30665.539**	**-30665.539**	**-30665.539**	**-30665.539**
g05	**5127.648**	5128.881	5174.492	5129.823
g06	**-6961.814**	-6875.940	-6961.284	-6737.877
g07	**24.316**	24.374	24.475	24.323
g08	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**
g09	**680.639**	680.665	680.643	680.646
g10	**7050.437**	7559.192	7253.047	7220.059
g11	**0.75**	**0.75**	**0.75**	**0.75**
g12	**-1**	**-1**	**-1**	**-1**
g13	**0.053950**	0.067543	0.166385	0.063862

Results on the Test Problems - Best

	ATMES	SRES	SMES	SRES-SC
g01	**-15**	**-15**	**-15**	**-15**
g02	-0.803388	-0.803515	-0.803601	-0.8032295
g03	**-1**	**-1**	**-1**	**-1**
g04	**-30665.539**	**-30665.539**	**-30665.539**	**-30665.539**
g05	**5126.498**	**5126.498**	5126.599	5126.512
g06	**-6961.814**	**-6961.814**	-6961.814	-6957.633000
g07	**24.306**	24.307	24.327	**24.306**
g08	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**
g09	**680.630**	**680.630**	680.632	**680.630**
g10	7052.253	7054.316	7051.903	**7050.189**
g11	**0.75**	**0.75**	**0.75**	**0.75**
g12	**-1**	**-1**	**-1**	**-1**
g13	**0.053950**	0.053957	0.053986	0.053988

Results on the Test Problems - Worst

	ATMES	SRES	SMES	SRES-SC
g01	**-15**	**-15**	**-15**	**-15**
g02	-0.756986	-0.726288	-0.751322	**-0.759766**
g03	**-1**	**-1**	**-1**	**-1**
g04	**-30665.539**	**-30665.539**	**-30665.539**	**-30665.539**
g05	**5135.256**	5142.472	5304.167	5149.931
g06	**-6961.814**	-6350.262	-6952.482	-6024.792
g07	**24.539**	24.642	24.483	24.395
g08	**-0.095825**	**-0.095825**	**-0.095825**	**-0.095825**
g09	**680.673**	680.763	680.719	680.725
g10	**7560.224**	8835.655	7638.366	7769.887
g11	**0.75**	**0.75**	**0.75**	**0.75**
g12	**-1**	**-1**	**-1**	**-1**
g13	**0.053999**	0.216915	0.4689294	0.157467

Intermediate Conclusion

- ATMES is the best
- SRES- SC (synthetic constraints) performs consistently better than SRES
- Note, however, ATMES has a very ad hoc mechanism for adjusting the threshold in converting equality constraints to inequality constraints

Design Optimization: Pressure Vessel

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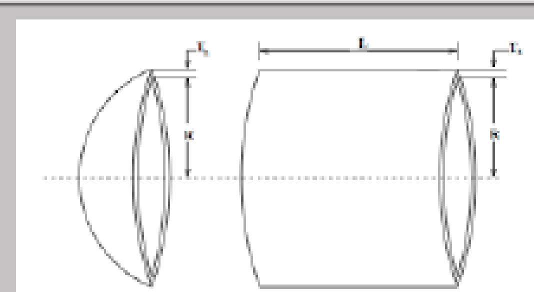
Pressure vessel design optimization problems

Bold Fonts: The best solutions

	Best	Mean	Worst
GA1	6288.745	6293.843	6308.150
GA2	6059.946	6177.253	6469.322
CAEP	NA	NA	NA
Mezura	6059.714	6379.938	NA
CPSO	6061.078	6147.133	6368.804
HPSO	6059.714	6099.932	6288.677
COPSO	6059.174	6071.013	NA
SiC-PSO	6059.714	6092.050	NA
NMPSO	5930.314	5946.790	**5960.056**
SRES-SC	**5885.333**	**5923.582**	6255.258



minimize

$$f(\vec{x}) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 x_4 + 19.84 x_1^2 x_3$$

subject to

$$g_1(\vec{x}) = -x_1 + 0.0193 x_3 \le 0$$

$$g_2(\vec{x}) = -x_2 + 0.00954 x_3 \le 0$$

$$g_3(\vec{x}) = \pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0$$

$$g_4(\vec{x}) = x_4 - 240 \le 0$$

Design Optimization – Speed Reducer

Speed reducer design optimization problems

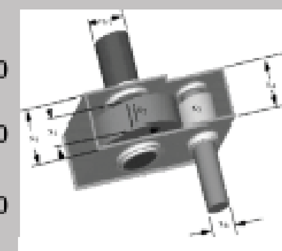
Bold Fonts: The best solutions

	Best	Mean	Worst
GA1	NA	NA	NA
GA2	NA	NA	NA
CAEP	NA	NA	NA
Mezura	2996.348	2996.348	NA
CPSO	NA	NA	NA
HPSO	NA	NA	NA
COPSO	2996.372	2996.409	NA
SiC-PSO	2996.348	2996.348	NA
NMPSO	NA	NA	NA
SRES-SC	**2996.231**	**2996.231**	**2996.231**

minimize

$$f(\vec{x}) = 0.7854x_1x_2^2(3.333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4x_6^2 + x_5x_7^2)$$

subject to



$$g_1(\vec{x}) = \frac{27}{x_1x_2^2x_3} - 1 \le 0$$

$$g_2(\vec{x}) = \frac{397.5}{x_1x_2^2x_3^2} - 1 \le 0$$

$$g_3(\vec{x}) = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \le 0$$

$$g_4(\vec{x}) = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \le 0$$

$$g_5(\vec{x}) = \frac{1.0}{110x_6^3}\sqrt{\left(\frac{745.0x_4}{x_2x_3}\right)^2 + 16.9 \times 10^6} - 1 \le 0$$

$$g_6(\vec{x}) = \frac{1.0}{85x_7^3}\sqrt{\left(\frac{745.0x_5}{x_2x_3}\right)^2 + 157.5 \times 10^6} - 1 \le 0$$

$$g_7(\vec{x}) = \frac{x_2x_3}{40} - 1 \le 0$$

$$g_8(\vec{x}) = \frac{5x_2}{x_1} - 1 \le 0$$

$$g_9(\vec{x}) = \frac{x_1}{12x_2} - 1 \le 0$$

$$g_{10}(\vec{x}) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0$$

$$g_{11}(\vec{x}) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0$$

Summary and Outlook

- Manipulating constraints to ease highly constrained optimization problems
- Incremental approximation of constraint functions
- Preliminary results suggest the idea is feasible
- More rigorous study is needed to verify the assumption that isolated feasible region is a result of complex constraints
- More specific test problems having isolated feasible regions should be constructed
- More sophisticated methods for manipulating the constraints are to be developed

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