Current trends in nonlinear optimisation

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The subject of optimisation is a fascinating blend of heuristics and rigour, of theory and experiment.

Roger Fletcher, 1987

Outline

- Current trends in nonlinear optimisation (my point of view)
- Our role in BtG
- Conic optimisation: what is it?
- A few applications

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Based on impressions from the 8th SIAM Conference on Optimisation, Boston, May 2008

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- derivative-free optimisation
- PDE constrained optimisation

- conic programming

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  nonlinear optimisation problems with integer (such as 0-1) variables. Highly demanded by practical applications and by the industry. Only a few pioneering codes exist so far, not yet ready for large-scale application. Due to the combinatorial nature of these problems, they have huge demands on computational resources.
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  Optimisation algorithms that do not require calculations of derivatives. Highly desirable in complex industrial applications. Many researchers work in this area, still the progress is slow and the current software is unable to solve even medium scale problems or problems with complex constraints.
- PDE constrained optimisation
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Our role in BtG

Mathematical optimisation should help with two goals:

- problem modelling
- solution algorithms and codes

Problem modelling

Find a computationally treatable and efficient formulation of the application problem.

The first, “rough” formulation of the problem is often numerically unsolvable. A few mathematical tricks can change it to a simple problem.

The users may not be aware of new optimisation problems and software.

Requires a tight cooperation of both parties.

Our role in BtG

Mathematical optimisation should help with two goals:

- problem modelling
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Solution algorithms and codes

Provide the user with a robust, efficient, well-documented software with user-friendly interface.

A lot of time and work has been invested in the developing of (general-purpose) optimisation software.

The potential users are often not aware of the software at all. Why?

- availability
- robustness (fine tuning of many parameters)
- documentation (often almost none)
- user-friendly interface

Optimisation software guides/resources

NEOS Server for Optimisation

http://www-neos.mcs.anl.gov/

Our optimisation solvers represent the state-of-the-art in optimisation software. Optimisation problems are solved automatically with minimal input from the user. Users only need a definition of the optimisation problem; all additional information required by the optimisation solver is determined automatically.

Optimisation software guide

http://www-fp.mcs.anl.gov/OTC/Guide/SoftwareGuide/

NEOS-Wiki

http://wiki.mcs.anl.gov/NEOS/index.php/NEOS_Wiki

The ubiquitous online source for optimisation (also known as mathematical programming, or numerical optimisation) and all its sub-disciplines, which replaces the NEOS guide.

Benchmarks for Optimisation Software

http://plato.la.asu.edu/bench.html

Conic programming

Generalization of classic nonlinear programming.

More and more practical problems are formulated in this way (structural optimisation, chemical engineering, machine learning, image recognition, etc).

Software in the early stages and the next years will see rapid development in this area.

Conic programming

Generalization of classic nonlinear programming:

Define $a \geq_K b$, for a, b vectors in $\mathbb{R}^n$, as

$$a - b \in K$$

where $K \subset \mathbb{R}^n$ is a pointed convex cone.

Example: $K = \mathbb{R}^n_+ := \{x \in \mathbb{R}^n \mid x_i \geq 0,\ i = 1, \ldots, n\}$
then $a \geq_K b$ is just the standard relation $a \geq b$.

$$\begin{array}{lcl} \min f(x) & & \min f(x) \\ \text{subject to} & \longrightarrow & \text{subject to} \\ \quad g_i(x) \geq 0,\ i = 1 \ldots, m & & \quad g_i(x) \geq_K 0,\ i = 1 \ldots, m \end{array}$$

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Two most prominent (non-trivial) cones:

- Lorentz cone $\longrightarrow$ second-order conic programming
- Cone of symmetric positive semidefinite matrices $\longrightarrow$ semidefinite programming

Basic reference:
A. Ben-Tal, A. Nemirovski: Lectures on Modern Convex Optimisation, SIAM, Philadelphia, 2001

Semidefinite programming—examples

Stability of matrices

A matrix $A \in \mathbb{M}^{k,l}$ is *stable* if all its eigenvalues have negative real parts.

Lyapunov criterion for stability: A is stable $\Leftrightarrow \exists P \in S^k$ such that

$$-A^T P - PA \succ 0, \qquad P \succ 0\,.$$

Formulated as SDP:

$$\begin{aligned} &\min_P \operatorname{tr}(P) \\ &\text{subject to} \\ &\qquad -A^T P - PA \succeq 0 \\ &\qquad P \succeq 0 \end{aligned}$$

Semidefinite programming—examples

The self-vibrations of a mechanical structure are the solutions of GEVP

$$K(x)w = \lambda M(x)w$$

$K(x)\ldots$ stiffness matrix

$M(x)\ldots$ mass matrix

$\lambda\ldots$ frequency

$w\ldots$ mode

Constraint $\lambda_{\min} \geq \hat{\lambda}$ equivalent to a conic constraint

$$K(x) - \hat{\lambda}M(x) \succeq 0$$

PLATO-N

A PLAtform for Topology Optimisation incorporating Novel, Large Scale, FMO methods

Academic project partners

- Technical University of Denmark (M. Bendsoe et. al.)
- Technion Haifa, Israel (A. Ben-Tal et. al.)
- University of Bayreuth, Germany (K. Schittkowski et. al.)
- University of Erlangen, Germany (G. Leugering, M. Stingl)
- University of Birmingham, UK (M. Kočvara et. al.)

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Industrial project partners

- Airbus UK Ltd.
- Altair Engineering Ltd., UK
- EADS Deutschland GmbH
- Eurocopter Deutschland GmbH
- RISC Software GmbH, Austria

Free Material Optimisation

Aim:

Given an amount of material, boundary conditions and external load f, find the material (distribution) so that the body is as stiff as possible under f.

The design variables are the material properties at each point of the structure.

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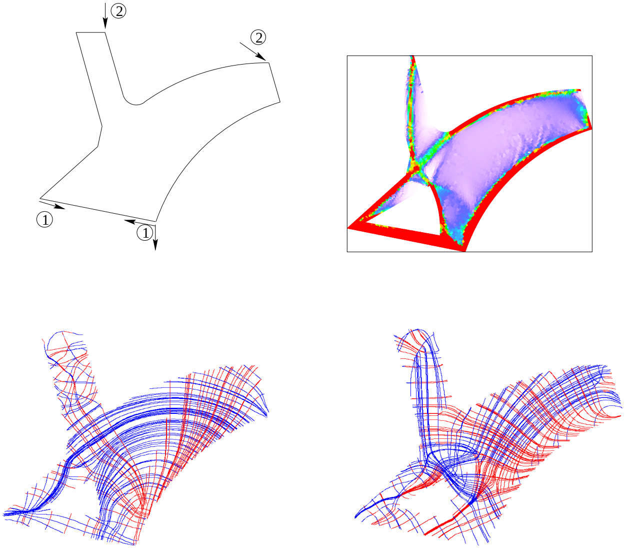
Bridging the gap between

- mechanical engineering
- material science
- mathematics

Free Material Optimisation



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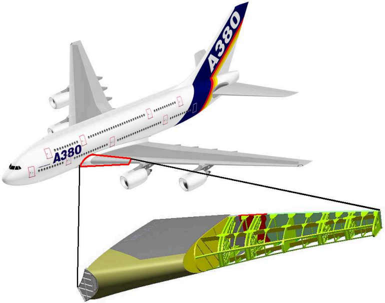

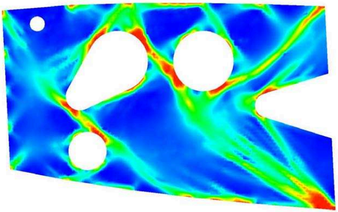

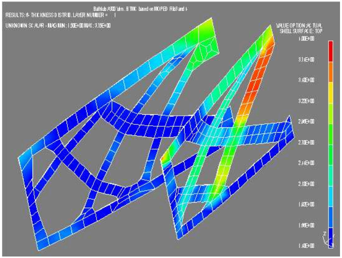

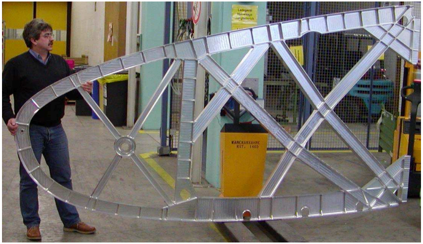

FMO SL primal formulation

Variable thickness sheet problem

$E = t\widehat{E}$

$\widehat{E} \ldots 3 \times 3$ symmetric matrix of material properties (given)

$t \ldots$ plate thickness (variable)

After discretization: $E_i = t_i\widehat{E}$

$$\min_{u\in\mathbb{R}^n, t\in\mathbb{R}^m} \sum_{i=1}^{m} t_i$$

subject to

$$0 \le t_i \le \bar{t}, \quad i = 1, \ldots, m$$

$$f^\top u \le \gamma$$

$$K(t)u = f$$

FMO SL primal formulation

FMO problem : the whole material matrix is the variable

Discretization: $E_1, \ldots, E_m$, $E_i \ldots 3 \times 3$ symmetric matrix

$$\min_{u \in \mathbb{R}^n, E_1, \ldots, E_m} \sum_{i=1}^{m} \mathrm{tr} E_i$$

subject to

$$E_i \succeq 0, \underline{t} \leq \mathrm{tr} E_i \leq \bar{t}, \quad i = 1, \ldots, m$$

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. . . nonlinear, non-convex semidefinite problem

PENNON

PENNON…software package for nonlinear conic optimisation problems.

Optimisation problems with nonlinear objective, nonlinear inequality and equality constraints, and semidefinite bounds:

$$\begin{aligned} \min_{x\in, Y_1\in\mathbb{S}^{p_1},\dots,Y_k\in\mathbb{S}^{p_k}} \quad & f(x,Y) && \text{(NLP-SDP)} \\ \text{subject to} \quad & g_i(x,Y)\le 0, && i=1,\dots,m_g \\ & h_i(x,Y)=0, && i=1,\dots,m_h \\ & \underline{\lambda}_i I \preceq Y_i \preceq \overline{\lambda}_i I, && i=1,\dots,k\,. \end{aligned}$$

Special versions for “pure” large-scale NLP, linear SDP, BMI

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The coming decades will be the decades of optimisation.

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